NB-15 (JEE) PHYSICS TWT ANS KEY & SOLUTIONS DT. 15-03-2023

: ANSWER KEY :

1)	d	2)	d	3)	b	4)	a
5)	a	6)	b	7)	d	8)	a
9)	c	10)	a	11)	a	12)	c
13)	d	14)	a	15)	c	16)	d
17)	b	18)	c	19)	c	20)	a
21)	2	22)	1	23)	1	24)	9
25)	2	26)	9	27)	3	28)	2
29)	3	30)	1				

: HINTS AND SOLUTIONS :

Single Correct Answer Type

1 **(d)**

law of conservation of momentum gives

$m_1 v_1 = m_2 v_2$

$\Rightarrow \frac{m_1}{m_2} = \frac{v_2}{v_1}$

But, $m = \frac{4}{3}\pi r^3 \rho$

or $m \propto r^3$

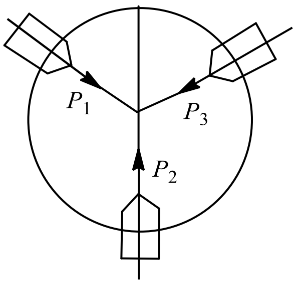
$\therefore \frac{m_1}{m_2} = \frac{r_1^3}{r_2^3} = \frac{v_2}{v_1}$

$\Rightarrow \frac{r_1}{r_2} = \left(\frac{1}{2}\right)^{1/3}$

$\therefore r_1 : r_2 = 1 : 2^{1/3}$

2 **(d)**

Three guns are fired towards the centre of circle as shown in figure.



Since, total final momentum is zero, and no external force is acting on the system, so total initial momentum should be also zero.

So, $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0$

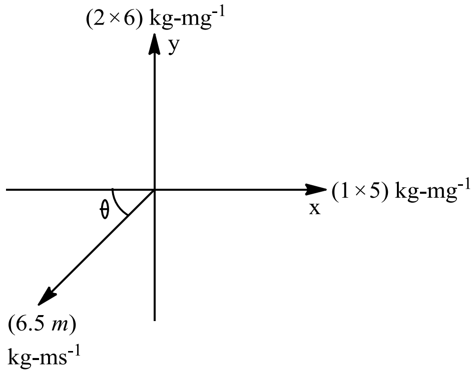
Three vectors, which are at an angle of $120°$ leads to zero resultant if and only if they have same magnitude.

So, $4.5 v_1 = 2.5 \times 575 = 4.5 v_2$

After solving, we will get v_1 and v_2 come out be 320 ms^{-1}.

3 **(b)**

We can realize the situation as shown in figure. Let the direction of mass m makes angle θ with the z-axis. Resolve momentum 6.5m along x and y –axis and equate.



Therefore, $6.5 m \cos\theta = 5 \times 1$

and $6.5\, m \sin\theta = 6 \times 2$

or $(6.5m)^2 = (5)^2 + (12)^2 = (13)^2$

or $m = \frac{13}{6.5} = 2 \text{ kg}$

therefore, total mass of the shell

$= 1 + 2 + 2 = 5$ kg

4 **(a)**

Let v be the velocity of the boat with respect to the water, then from conservation of linear momentum.

$(200 + 50)v + 50 \times 2 = 50 \times 0 + 200 \times 0$

$250v = -100$

$v = -\frac{100}{250} = -\frac{2}{5} \text{ ms}^{-1}$

5 **(a)**

Since, $F = \frac{\Delta p}{\Delta t}$

or $\Delta p = F\, \Delta t$

We can say that momentum between 0 to 7 s is equal to the vector area enclosed by the force-time graph from 0 to 7 s. So, Change in linear momentum

= vector area of triangle *OAB* + vector area of square BCDE+ vector area of triangle EFG + vector area of square *GHIJ* + vector area of triangle *JKL*

$$= \left[\frac{1}{2} \times 1 \times (-1)\right] + [2 \times 2] + \left[\frac{1}{2} \times 2 \times (-2)\right] + [1 \times 1] + \left[\frac{1}{2} \times 1 \times (-1)\right]$$

$$= -\frac{1}{2} + 4 - 2 + 1 - \frac{1}{2} = 2 \text{ Ns}$$

6 **(b)**

For a given mass $P \propto v$. If the momentum is constant then its velocity must be constant

7 **(d)**

Rate of flow of water $\frac{V}{t} = \frac{10\, cm^3}{\text{sec}} = 10 \times 10^{-6} \frac{m^3}{\text{sec}}$

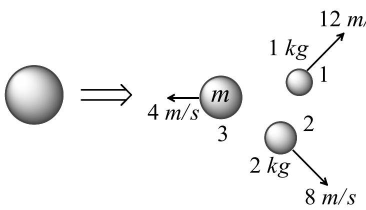
Density of water $\rho = \frac{10^3 kg}{m^3}$

Cross-sectional area of pipe $A = \pi(0.5 \times 10^{-3})^2$

Force $= m\frac{dv}{dt} = \frac{mv}{t} = \frac{V\rho v}{t} = \frac{\rho V}{t} \times \frac{V}{At} = \left(\frac{V}{t}\right)^2 \frac{\rho}{A} \left(\because v = \frac{V}{At}\right)$

By substituting the value in the above formula we get $F = 0.127\, N$

8 **(a)**



According to conservation of linear momentum

$p_3 = \sqrt{p_1^2 + p_2^2}$

$\Rightarrow m \times 4 = \sqrt{(1 \times 12)^2 + (2 \times 8)^2} = 20 \Rightarrow m = 5\, kg$

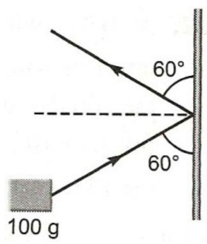
9 **(c)**

Change of momentum **F** $\Delta t = m\Delta \mathbf{v}$

$\Rightarrow \mathbf{F} = \frac{m\Delta \mathbf{v}}{t}$

By doing so time of change in momentum increases and impulsive force on knees decreases.

10 **(a)**



Change in the velocity $= v \sin\theta - (-v\sin\theta) = 2\sin\theta$

Change in the momentum

$\Delta p = 2\, m\, v \sin\theta$

$\therefore$ Force applied $F = \frac{\Delta p}{\Delta t}$

$= \frac{2 \times 100 \times 10^{-3} \times 5 \sin\theta\, 60°}{2 \times 10^{-3}}$

$= 100 \times 5 \times \frac{\sqrt{3}}{2}$

$= 250\sqrt{3}$ N (To the right)

11 **(a)**

$y_{\text{CM}} = \frac{m_1y_1+m_2y_2+m_3y_3+m_4y_4+m_5y_5}{m_1+m_2+m_3+m_4+m_5}$

$= \frac{(6m)(0)+m(a)+m(a)+m(0)+m(-a)}{6m+m+m+m+m} = \frac{a}{10}$

12 **(c)**

At the time of maximum compression, the speeds of blocks will be the same. Let that speed be v and maximum compression be x

Applying conservation of momentum,

$(m_1 + m_2)v = m_1v_1 + m_2v_2$

$\Rightarrow v = 4$ m/s

Applying conservation of mechanical energy

$\frac{1}{2}kx^2 + \frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

Solving, we get $x = 0.02$ m

13 **(d)**

$u_x = 20\sqrt{2}\cos 45° = 20$ m/s

$u_y = 20\sqrt{2}\, \sin 45° = 20$ m/s

After 1 s, horizontal component remains unchanged while vertical component becomes

$v_y = u_y - \text{g}t = 20 - 10 = 10$ m/s

Due to explosion, one part comes to rest. Hence, from conservation of linear momentum, vertical component of second part will become $v_{y1} = 20$ m/s. Therefore, maximum height attained by the second part will be

$H = h_1 + h_2$

Here, h_1 = height attained in 1 s

$= (20)(1) - \frac{1}{2} \times 10 \times 1^2 = 15\text{ m}$

and h_2 = height attained after 1 s

$\frac{v_{y1}^2}{2\text{g}} = \frac{(20)^2}{2 \times 10} = 20\text{ m}$

$\therefore H = 15 + 20 = 35\text{ m}$

14 **(a)**

Applying the law of conservation of momentum,

$m_1v_1 = (m_1 + m_2)V$ (i)

Where $v_1 = \sqrt{2\text{g}d}$ is the velocity with which m_1 collides with m_2

Therefore,

$V = \frac{m_1}{(m_1 + m_2)}\sqrt{2\text{g}d}$

Now, let the centre of mass rise through a height h after collision. In this case, the kinetic energy of $m_1 + m_2$ system is converted into potential energy at maximum height h

$$\Rightarrow \frac{1}{2}(m_1 + m_2)V^2 = (m_1 + m_2)gh$$

$$\Rightarrow \frac{1}{2}(m_1 + m_2)\left\{\frac{m_1}{m_1 + m_2}\right\}^2 2gd = (m_1 + m_2)gh$$

$$\Rightarrow h = d\left\{\frac{m_1}{m_1 + m_2}\right\}^2$$

15 **(c)**

$mv = mv_1 + nmv_2$

$v = v_1 + nv_2$

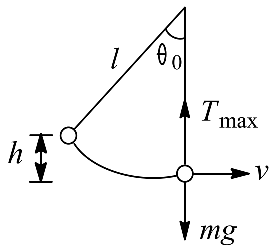
$= v_2\ v_1$

$$\Rightarrow v_2 = \frac{2v}{(n+1)}, v_1 = \left(\frac{1-n}{1+n}\right)v$$

$$\frac{\mathrm{KE}_1}{\mathrm{KE}_2} = \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \left(\frac{n-1}{n+1}\right)^2$$

16 **(d)**

Maximum tension in the string is in its lowest position. Speed of mass m in its lowest position is



$v^2 = 2gh = 2gl(1 - \cos\theta_0)$

$$T_{\max} - mg = \frac{mv^2}{l}$$

$T_{\max} = mg + 2\,mg(1 - \cos\theta_0)$

$= mg\,(3 - 2\cos\theta_0)$

Block of mass 4m dose not move. So, $\mu(4mg) \geq T_{\max}$

Or $4\,\mu\, mg \geq mg\,(3 - 2\cos\theta_0)$

Or $\mu \geq \left(\frac{3-2\cos\theta_0}{4}\right)$

17 **(b)**

$$K = \frac{P^2}{2m}$$

From the graph, $4 = \frac{4^2}{2m} \Rightarrow m = 2\ \mathrm{kg}$

18 **(c)**

Neglecting gravity

$$v = u\ln\left(\frac{m_0}{m_1}\right)$$

$u =$ ejection velocity w.r.t. balloon

$m_0 =$ initial mass

$m_t =$ mass at any time t

$$v = 2\ln\left(\frac{m_0}{m_0/2}\right) = 2\ln 2\ \mathrm{m/s}$$

19 **(c)**

$$\vec{v}_{\mathrm{CM}} = \frac{m_1v_1 + m_2v_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{6mv}{7m} = \frac{6v}{7}$$

20 **(a)**

Because the collision is perfectly inelastic, hence the two blocks stick together. By conservation of linear momentum,

$$2mV = mv \text{ or } V = \frac{v}{2}$$

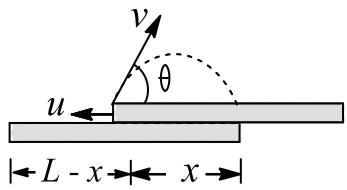
By conservation of energy,

$2mgh = \frac{1}{2}2mV^2$ or $h = v^2/8g$

Integer Answer Type

21 **(2)**

Apply conservation of momentum in horizontal direction:



$mv\cos\theta - \mathrm{mu} = 0 \Rightarrow u = v\cos\theta$

$L - x = ut, x = v\cos\theta t$

Solve to get, $x = \frac{L}{2}$

$$x = \frac{v^2\sin 2\theta}{g} \Rightarrow \frac{L}{2} = \frac{v^2\sin 2\theta}{g}$$

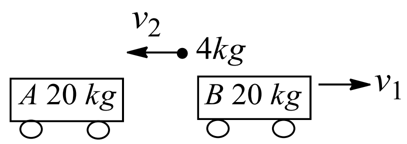
$\Rightarrow v = \sqrt{\frac{gL}{2\sin\theta}}$ for minimum $v, \sin 2\theta = 1$

$v_{\min} = \sqrt{\frac{gL}{2}} = \sqrt{\frac{10\times5}{2}} = 2\sqrt{5}\ \mathrm{m/s}$, Hence n =2

22 **(1)**

All the velocities shown in diagrams are w.r.t. ground

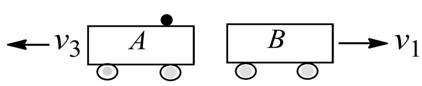
After first jump:



$20v_1 = 4v_2$ and $v_1 + v_2 = 6$ (given)

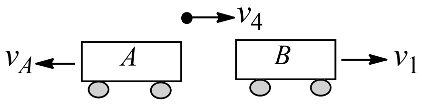
Solve to get $v_1 = 1\ \mathrm{m/s}, v_2 = 5\ \mathrm{m/s}$

When child arrives on A:



$(20 + 4)v_3 = 4v_2 \Rightarrow v_3 = 5/6\ \mathrm{m/s}$

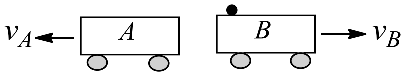
After the second jump:



$v_4 + v_A = 6, 24v_3 = 20v_A - 4v_4$

Solve to get $v_A = \frac{11}{6}\text{m/s}, v_4 = \frac{25}{6}\text{m/s}$

When child arrives on B:



$24v_B = 4v_4 + 20v_1$

$\Rightarrow\ 24v_B = 4\left(\frac{25}{6}\right) + 20 \times 1 \Rightarrow v_B = \frac{55}{36}$ m/s

Now $\frac{6v_B}{5v_A} = \frac{6\times55\times6}{36\times5\times11} = 1$

23 **(1)**

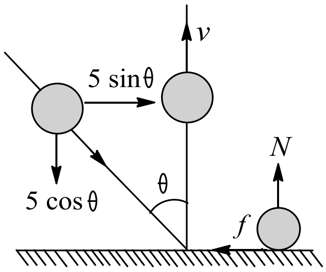
From impulse-momentum theorem,

$\int N\,dt = m(v + 5\cos\theta)$ (i)

$\int f\,dt = m5\sin\theta$

$\mu\int N\,dt = m5\sin\theta$ (ii)

$\Rightarrow\ \mu m(v + 5\cos\theta) = m5\sin\theta$



According to Newton's law of restitution,

$v = e\,5\cos\theta$

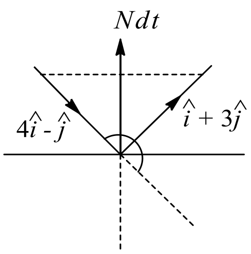
Solve to get $\mu = 1$

24 **(9)**

$\int Ndt = 1[(\hat{\imath} + 3\hat{\jmath}) - (4\hat{\imath} - \hat{\jmath})] = -3\hat{\imath} + 4\hat{\jmath}$

Component of $4\hat{\imath} - \hat{\jmath}$ along $-3\hat{\imath} + 4\hat{\jmath}$

$= \frac{-12-4}{25}(-3\hat{\imath} + 4\hat{\jmath}) = -\frac{16}{25}(-3\hat{\imath} + 4\hat{\jmath})$



Speed of approach$= \frac{16}{25}\sqrt{25} = \frac{16}{5}$

Component of $\hat{\imath} + 3\hat{\jmath}$ along $-3\hat{\imath} + 4\hat{\jmath}$ is $\frac{-3+12}{25}(-3\hat{\imath} + 4\hat{\jmath})$

Speed of separation = 9/5

Speed of separation= e ×speed of approach

$e = \frac{9}{16}$ $n = 9$

25 **(2)**

$F_1 = 4N = -4\hat{\imath}$ $0 < t \le 1$ s

$= 2N = -2\hat{\imath}$ $1 \le t \le 3$ s

$F_2 = 1N = -\hat{\jmath}$ $0 < t \le 2$ s

$= 2t - 3N = -(2t-3)\hat{\jmath}$ $2 \le t \le 3$ s

Initial velocity f the particle is $\vec{u} = 10\hat{\imath}$

From impulse momentum theorem, $\int d\vec{p} = \int \vec{F}\,dt$

$m\vec{v} - m\vec{u} = \int^3 (\vec{F}_1 + \vec{F}_2)dt$

(Where $\vec{v}$ is the required velocity)

$1 \times \vec{v} - 1 \times 10\hat{\imath} = -8\hat{\imath} - 4\hat{\jmath}$ or $\vec{v} = 2\hat{\imath} - 4\hat{\jmath}$

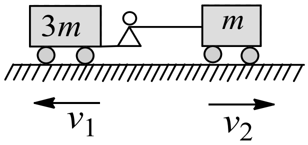
Or $v = \sqrt{2^2 + 4^2} = 2\sqrt{5}$ m/s

26 **(9)**

Trolleys gain momentum due to force applied by man which will be internal force for the system of trolleys and man and there is no other external force. Here we assume that man applies force for a very short time, during which effect of friction can be neglected.

Momentum just before pushing = momentum just after pushing

$0 = 3\,mv_1 - mv_2 \Rightarrow v_1 = \frac{v_2}{3}$



From work-energy theorem for individual trolleys,

$f_1S_1 = \frac{1}{2}3mv_1^2, f_2S_2 = \frac{1}{2}3mv_2^2$

Here $f_1 = \mu\,3mg, f_2 = \mu mg$

Solve to get $\frac{S_2}{S_1} = \left(\frac{v_2}{v_1}\right)^2 = 9$

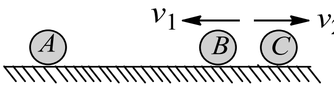
27 **(3)**

To save himself, the man throws his jacket in opposite direction to the lake According to momentum conservation, he himself gets a velocity in the direction of the lake. During the motion as gravity is the only external force on the system (man plus jacket), centre of mass will not be displaced horizontally. Thus, centre of mass of the system falls vertically and when the man falls in the lake, jacket falls at a point such that the centre of mass of the man and the jacket will be

directly below the point from where the man jumps)
As it is given that man falls at a distance d from this point, it implies that jacket will fall at a distance x in the opposite direction such that $mx = Md \Rightarrow x = \frac{M}{m}d = 29\text{ m}$, man has to travel a distance $x + d = 29 + 1 = 30$ m to pick his jacket

28 **(2)**

For the first collision, $e = 1, v = v_1 + v_2$



$\Rightarrow \quad v_2 = v - v_1$ (i)

By momentum conservation

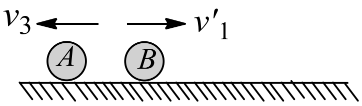
$m_B v = -m_B v_1 + m_C v_2$

$m_B v = -m_B v_1 + 4\, m_B v_2$

$v_2 = \frac{v_1 + v}{4}$ (ii)

From Eqs. (i) and (ii), $v_1 = \frac{3}{5}v$ and $v_2 = \frac{2}{5}v$

For the second collision, $e = 1$



$v_1 = v_1' + v_3 \Rightarrow v_3 = v_1 - v_1'$ (iii)

By momentum conservation, $-m_B v_1 = m_B v_1' - m_A v_3$

Or $-m_B v_1 = m_B v_1' - 4\, m_B v_3$ $(\because m_A = 4m_B)$

$v_3 = \frac{v_1' + v_1}{4}$ (iv)

From Eqs. (iii) and (iv), $v_1' = \frac{3}{5}\, v_1 = \frac{3}{5}\left(\frac{3}{5}v\right) = \frac{9}{25}v$

Clearly, $\frac{9}{25}\, v < \frac{2}{5}v$

Therefore, 'B' cannot collide with 'C' for the second time

Hence, the total number of collisions is 2

29 **(3)**

For first case: $W = \Delta\text{KE} \Rightarrow -Rd = -\frac{1}{2}mu^2$

$Rd = \frac{1}{2}mu^2$ (i)

For second case: $W = (M + m)v, v$ is common velocity

$$\Rightarrow v = \frac{m}{(M + m)}u$$

$$-Rd' = \frac{1}{2}(M + m)v^2 - \frac{1}{2}mu^2 = \frac{1}{2}\frac{m^2u^2}{(M + m)} - \frac{1}{2}mu^2$$

$\Rightarrow \quad Rd' = \frac{1}{2}\frac{Mm}{(M+m)}u^2$ (ii)

Solving Eqs. (i) and (ii), $\frac{Rd}{Rd'} = \frac{M+m}{M} \Rightarrow d' = 3\text{ cm}$

30 **(1)**

Let u be the initial velocity of the ball of mass m. Then

$mu = mv_1 + nmv_2 \Rightarrow v_1 + nv_2 = u$ (i)

For elastic collision, Newton's experimental formula is $(u_2 = 0)$

$v_1 - v_2 = -(u_1 - u_2) \Rightarrow v_1 - v_2 = -u$ (ii)

Solving Eqs. (i) and (ii), $v_1 = \frac{1-n}{1+n}u$

Fractional loss in KE (= fractional transfer of KE)

$$f = \frac{K_i - K_f}{K_i} - \frac{\frac{1}{2}mu^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mu^2} = 1 - \left(\frac{v_1}{u}\right)^2$$

$$f = 1 - \left(\frac{1 - n}{1 + n}\right)^2 = \frac{4n}{(n + 1)^2}$$

The transfer of energy is maximum when $f = 1$ or 100%

$$\frac{4n}{(n + 1)^2} = 1 \Rightarrow n = 1$$

This is, the transfer of energy is maximum when the mass ratio is unity